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TEACHING MATHS



Topic 10

Probability, Permutation and Combination
概率、排列與組合



Focus Points 本課重點

- What is Probability (F.3) 什麼是概率
- Mutually Exclusive Events (F.3) 互斥事件
- Independent Events (F.3) 獨立事件
- Dependent Events (F.3) 相關事件
- Conditional Probability (F.5) 條件概率
- Expected Values (F.3) 期望值
- Set Theory & Venn Diagram (F.5) 集合論及溫氏圖
- Permutation (F.5) 排列
- Combination (F.5) 組合



What is Probability

什麼是概率



What is Probability 什麼是概率

- Describing the change/likelihood that an event (事件) will occur
- Probability must be between 0 and 1
- Basic Formula: Probability = $\frac{\text{No. of Occurance of Desired Events}}{\text{No. of Occurance of All Events}}$



Mutually Exclusive Events

互斥事件



Mutually Exclusive Events 互斥事件

- If Event A occurs, then Event B must not occur
- Example 1: Event(Male) and Event(Female)
- Example 2: For a dice, Event (1) and Event (not 1)
- Example 3: For poker, Event (diamond 3) and Event (not diamond 3)
- Sum of Probabilities of Mutually Exclusive Events must be 1



Independent Events

獨立事件



Independent Events 獨立事件

- The occurrence of an event would not be affected by the previous result.
- Example 1: Throw a dice to get a “2”
- Example 2: The way you win Mark Six
- Example 3: Draw a white ball in a bag containing balls with different colours when you put the drawn ball back to the bag every time
- Independent event and Dependent event are mutually exclusive.
- $P(A \text{ and } B) = P(A) \times P(B)$



Dependent Events 相關事件



Dependent Events 相關事件

- The occurrence of an event would be affected by the previous result.
- Example 1: Draw a white ball in a bag containing balls with different colours when you don't put the drawn ball back to the bag
- Example 2: Draw 2 notes from your wallet given that you have \$20, \$100 and \$500 notes
- $P(A \text{ and } B) = P(A) \times P(B|A)$

Probability of B given A



Conditional Probability

條件概率



Conditional Probability 條件概率

- The condition(s) limit(s) the choice of events
- Example 1: Given that the dice is thrown and an even number is out, what is the probability of “2” is out?
- Example 2: A bag contains 3 white balls and 1 black ball. Given that a black ball was drawn and no replacement was done, what is the probability of drawing a black ball?
- $P(A \text{ given } B) = \frac{P(A \text{ and } B)}{P(B)}$



Expected Value 期望值



Expected Value 期望值

- Similar concept to Weighed Mean
- Sum of all events with respect to their probabilities
- Example: The expected score of a MC test if you randomly guess all the questions
- Expected Value = $x_1P(1) + x_2P(2) + \dots + x_nP(n)$



Set Theory and Venn Diagram

集合論及溫氏圖

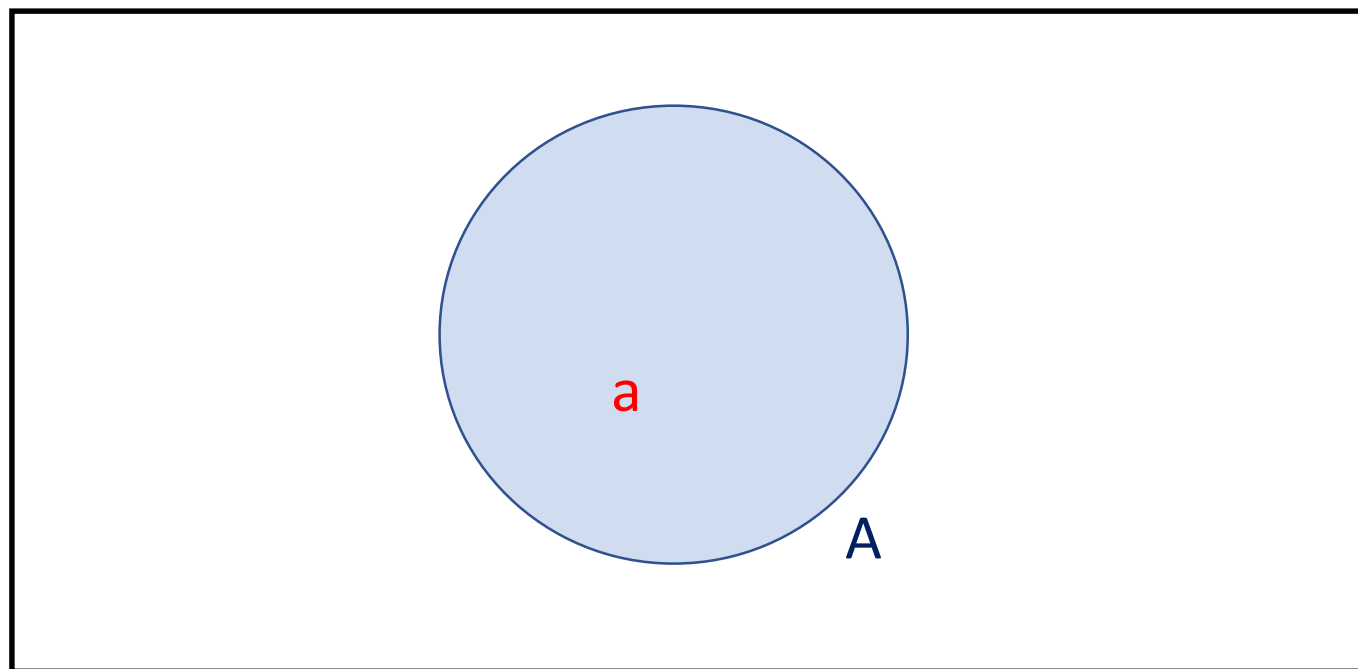


Set Theory 集合論

- Notations in set theory:
 - $A = \{a,b,c,d\}$: Set A contains elements a,b,c,d
 - $a \in A$: a is belong to set A / a is an element of set A
 - $A \subset B$: A is a subset of B
 - \emptyset : Empty set
 - \cup : Or / Union
 - \cap : And / Intersection
 - A' : Converse of set A / Complement of set A
 - $B \setminus A$: Converse of set A in B / Complement of set A in B

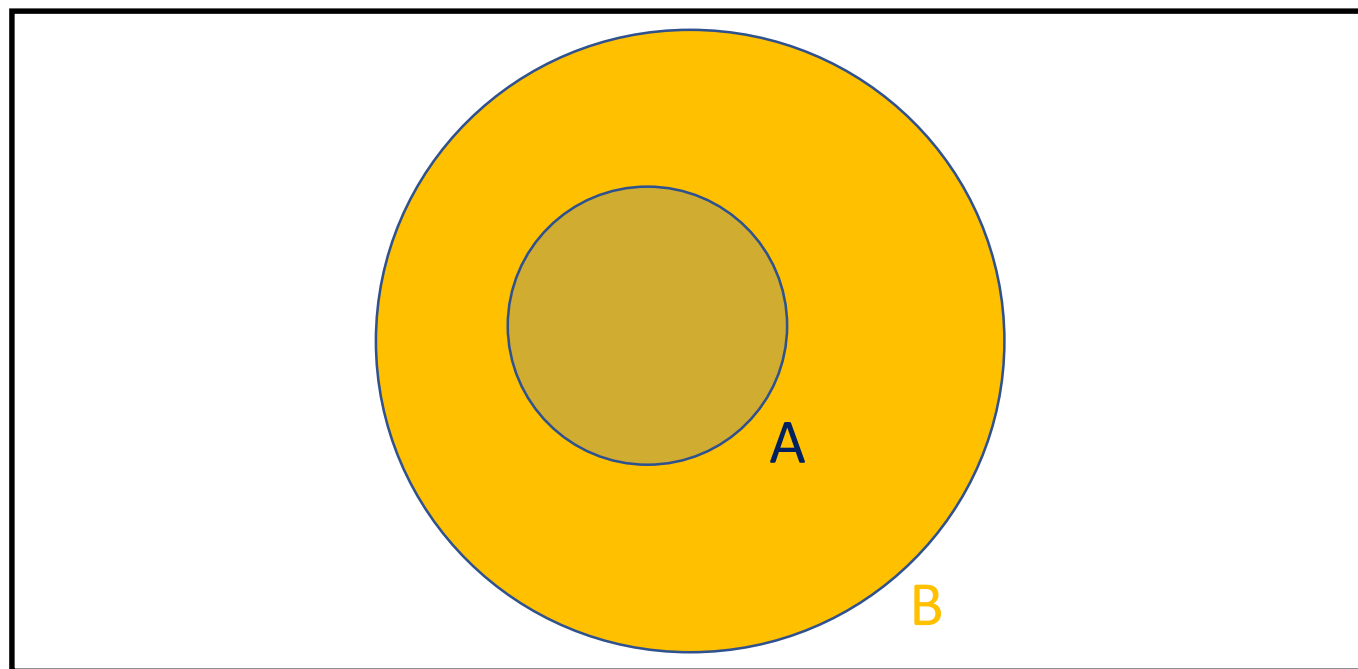
Venn Diagrams 溫氏圖

- $a \in A$: a is belong to set A / a is an element of set A



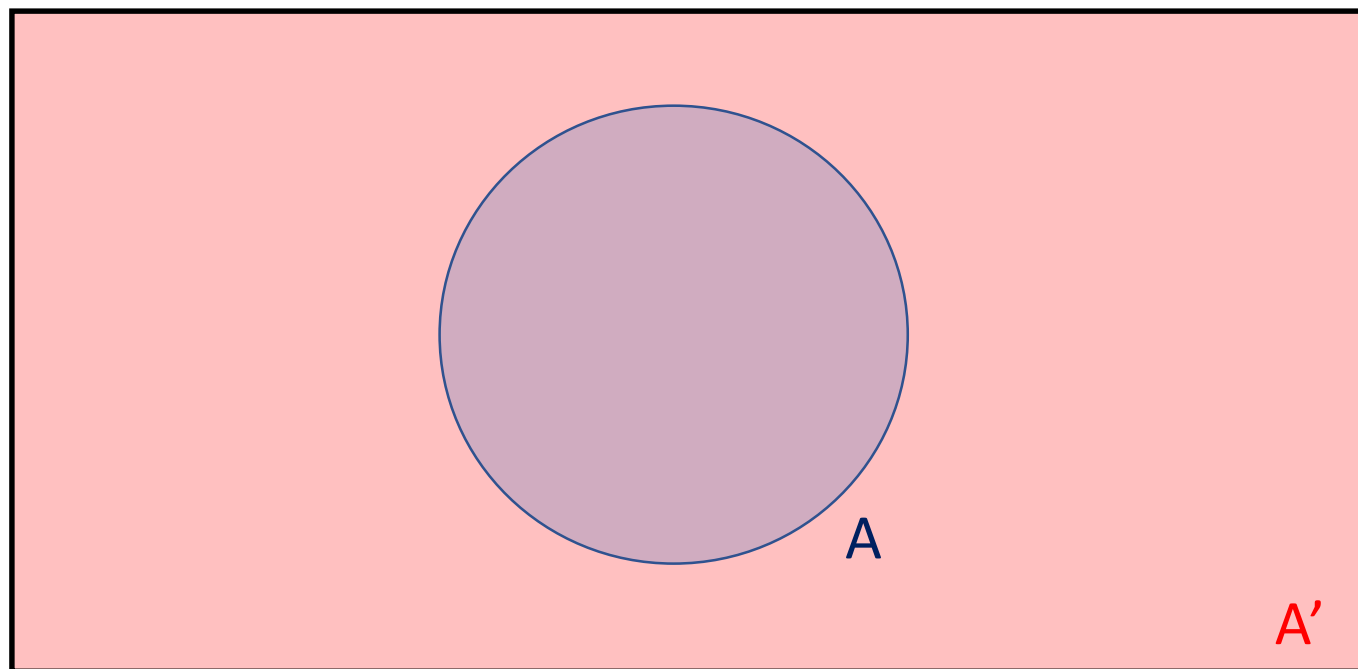
Venn Diagrams 溫氏圖

- $A \subset B$: A is a subset of B



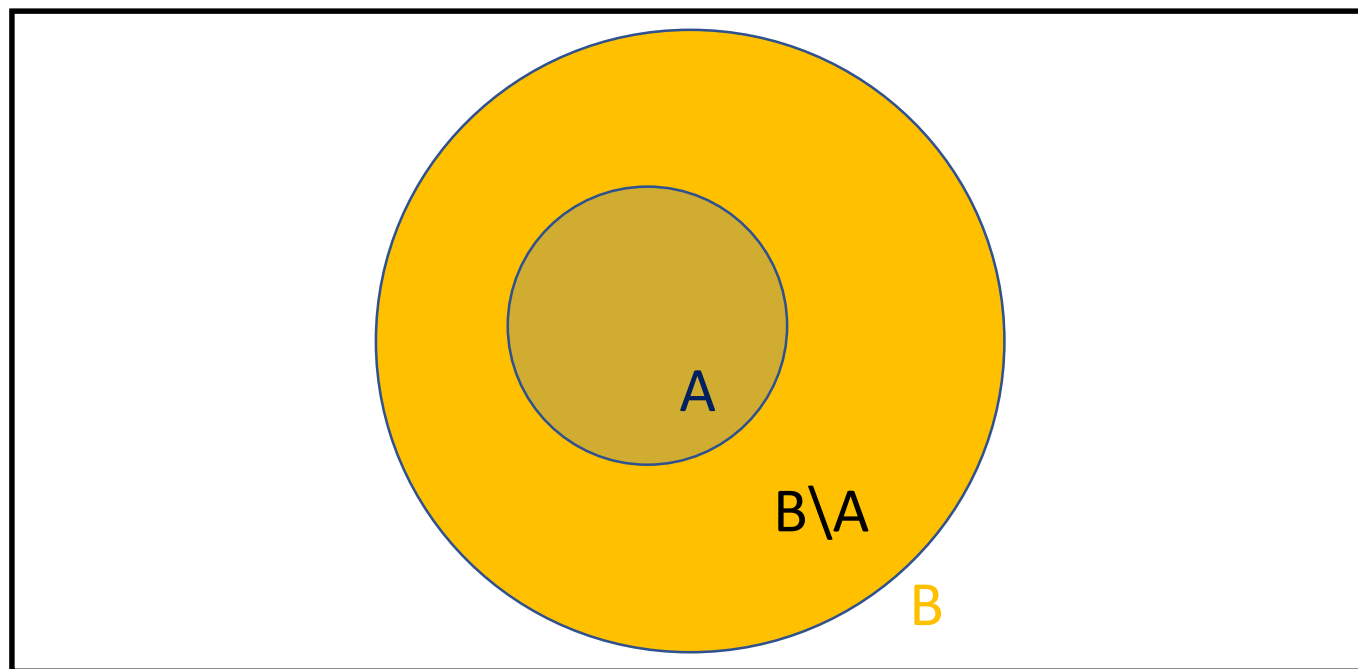
Venn Diagrams 溫氏圖

- A' : Converse of set A / Complement of set A



Venn Diagrams 溫氏圖

- $B \setminus A$: Converse of set A in B / Complement of set A in B





Practice 1

練習 1



HKDSE 2012 Maths Paper 2 Q27

9★◆ is a 3-digit number, where ★ and ◆ are integers from 0 to 9 inclusive. Find the probability that the 3-digit number is divisible by 5.

A. $\frac{1}{5}$

B. $\frac{7}{33}$

C. $\frac{20}{99}$

D. $\frac{19}{100}$

HKDSE 2012 Maths Paper 2 Q28



The stem-and-leaf diagram below shows the distribution of the ages of a group of members in a recreational centre.

<u>Stem (tens)</u>	<u>Leaf (units)</u>
5	0 5 6 6 8
6	1 4 5 5 7 8 8 9
7	3 4 4 6 7 9
8	
9	1

A member is randomly selected from the group. Find the probability that the selected member is not under the age of 74.

- A. 0.2
- B. 0.3
- C. 0.7
- D. 0.8



HKDSE 2012 Maths Paper 2 Q44

A box contains six balls numbered 7, 8, 8, 9, 9 and 9 respectively. John repeats drawing one ball at a time randomly from the box without replacement until the number drawn is 9. Find the probability that he needs exactly three draws.

- A. $\frac{1}{2}$
- B. $\frac{1}{6}$
- C. $\frac{1}{8}$
- D. $\frac{3}{20}$



HKDSE 2014 Maths Paper 2 Q27

A box contains m yellow balls and 20 black balls. If a ball is randomly drawn from the box, then the probability of drawing a yellow ball is $\frac{1}{m}$. Find the value of m .

- A. 4
- B. 5
- C. 15
- D. 25



HKDSE 2018 Paper 2 Q28

Two numbers are randomly drawn at the same time from seven cards numbered 1, 1, 1, 2, 2, 3 and 4 respectively. Find the probability that the sum of the numbers drawn is 5.

A. $\frac{5}{21}$

B. $\frac{5}{42}$

C. $\frac{5}{49}$

D. $\frac{10}{49}$



HKDSE 2013 Paper 1 Q10

The ages of the members of Committee A are shown as follows:

17	18	21	21	22	22	23	23	23	31
31	34	35	36	47	47	58	68	69	69

- (a) Write down the median and the mode of the ages of the members of Committee A . (2 marks)
- (b) The stem-and-leaf diagram below shows the distribution of the ages of the members of Committee B . It is given that the range of this distribution is 47.

<u>Stem (tens)</u>	<u>Leaf (units)</u>
2	a 5 6 7
3	3 3 8
4	3
5	1 2 9
6	7 b

- (i) Find a and b .
- (ii) From each committee, a member is randomly selected as the representative of that committee. The two representatives can join a competition when the difference of their ages exceeds 40. Find the probability that these two representatives can join the competition.

(4 marks)



HKDSE 2013 Paper 1 Q16

A box contains 5 white cups and 11 blue cups. If 6 cups are randomly drawn from the box at the same time,

- (a) find the probability that at least 4 white cups are drawn; (2 marks)
- (b) find the probability that at least 3 blue cups are drawn. (2 marks)



HKDSE 2015 Paper 1 Q16

A box contains 5 red bowls, 6 yellow bowls and 3 white bowls. If 4 bowls are randomly drawn from the box at the same time,

- (a) find the probability that exactly 2 red bowls are drawn; (2 marks)
- (b) find the probability that at least 2 red bowls are drawn. (2 marks)

HKDSE 2014 Paper 1 Q19



Ada and Billy play a game consisting of two rounds. In the first round, Ada and Billy take turns to throw a fair die. The player who first gets a number '3' wins the first round. Ada and Billy play the first round until one of them wins. Ada throws the die first.

- (a) Find the probability that Ada wins the first round of the game. (3 marks)
- (b) In the second round of the game, balls are dropped one by one into a device containing eight tubes arranged side by side (see Figure 8). When a ball is dropped into the device, it falls randomly into one of the tubes. Each tube can hold at most three balls.

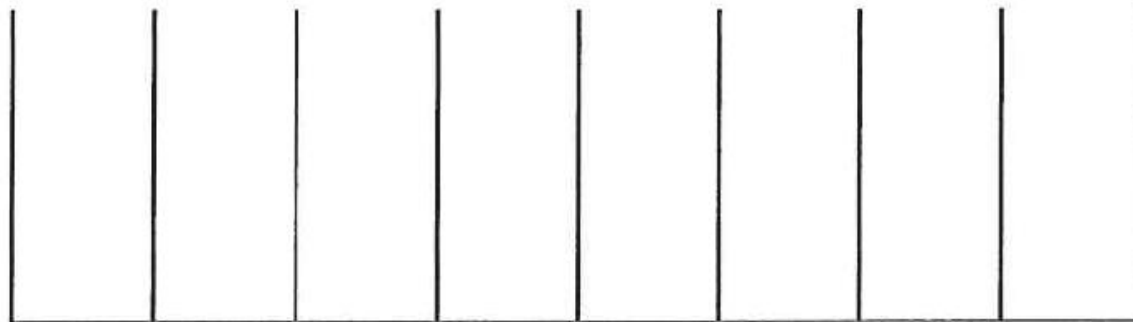


Figure 8



The player of this round adopts one of the following two options.

- Option 1: Two balls are dropped one by one into the device. If the two balls fall into the same tube, then the player gets 10 tokens. If the two balls fall into two adjacent tubes, then the player gets 5 tokens. Otherwise, the player gets no tokens.
- Option 2: Three balls are dropped one by one into the device. If the three balls fall into the same tube, then the player gets 50 tokens. If the three balls fall into three adjacent tubes, then the player gets 10 tokens. If the three balls fall into two adjacent tubes, then the player gets 5 tokens. Otherwise, the player gets no tokens.
- (i) If the player of the second round adopts Option 1, find the expected number of tokens got.
- (ii) Which option should the player of the second round adopt in order to maximise the expected number of tokens got? Explain your answer.
- (iii) Only the winner of the first round plays the second round. It is given that the player of the second round adopts the option which can maximise the expected number of tokens got. Billy claims that the probability of Ada getting no tokens in the game exceeds 0.9. Is the claim correct? Explain your answer.

(10 marks)